Accounting for subsistence needs in cost-benefit analysis

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Abstract

Revealed and stated preference techniques are widely used to assess non-market components, in particular in cost-benefit analysis (CBA). First, however, individuals have to satisfy subsistence needs through market good consumption, which affects their ability to pay. We provide a methodological framework and derive a correction factor to account for this effect. We quantify the impacts of neglecting it on the desirability and the ranking of projects from a theoretical, a numerical and an empirical perspective. A plutocratic bias emerges: the views of the richest - whatever they are - are more likely to impact CBA-based decision-making.

1 Introduction

Over the last century, cost-benefit analyses (CBA) have increasingly been used in all economic sectors to support public and private decision-making. The past 50 years have seen non-market components break into CBA, with considerations like improved recreation, visual amenities, odours, noise, loss of biodiversity, psychological factors, or the valuation of health. However, with no marketplace to set economic prices for these components, their value has to be assessed by methods based on stated or revealed preferences. These valuation methods elicit individuals' preferences for a given non-market good or service, either directly through surveys (stated) or indirectly through data collection (revealed). They then derive willingness to pay (WTP) for the corresponding welfare change. After statistical treatment, this WTP is used by private and public decision-makers in CBA. Thus, willingness to exchange or receive money for non-market goods or service provision acts as an indicator of public preferences.

This process appears very democratic, directly feeding the preferences of the whole population into public decision-making. Nevertheless, according to Pearce et al. (2006) "CBA tends to work with measures of benefit and cost based on willingness to pay which, in turn, is heavily influenced by ability to pay (income, wealth). The result is a cost-benefit rule for sanctioning or rejecting projects or policies that is biased in favour of those with higher incomes, raising issues of distributional fairness". For this reason, it is common to apply distributional weights, representing the social marginal utility of income, to individual WTP. However, income is not the only determinant of ability to pay: the irreducible costs of meeting subsistence needs (often pre-incurred) limit individuals' ability to pay. The literature contains some work that takes this into account by considering subsistence needs effects when measuring minimum health conditions (Russell, 1996) or minimum levels for ecosystem services (Baumgärtner et al., 2017b; Drupp, 2018). There have also been proposals for income-based correction to capture inequalities in ability to pay (Donaldson, 1999; Shono et al., 2014; Breffle et al., 2015). Surprisingly however, to our knowledge, the impact of individual subsistence needs on CBA has not been explored either theoretically or empirically.

We propose to fill this gap by investigating how individual subsistence needs affect collective choices made through CBA. First, we consider how subsistence needs impact individual WTP and the total benefits measured from WTP aggregation, and look for a relevant individual subsistence-needs correction factor. Second, we introduce heterogeneous preferences towards the non-market component, and examine how subsistence needs affect both the CBA outcome and the likelihood of ranking reversal between two projects with different non-market components. Finally, we provide both numerical and empirical illustrations for various non-market component preferences.

We find, as expected, that the WTP of the poorest individuals is more impacted by subsistence needs than that of the richest, which decreases the desirability of a project at the aggregate level. We show that assigning standard distributional weights based on the social marginal utility of income does not correct for the effect of subsistence needs. When a population's preferences are heterogeneous, the views of the richest - whatever they are - are more likely to impact decision-making based on CBA: this can be considered a plutocratic bias. We determine the conditions under which ranking reversal between two projects will occur, depending on the preferences of the richest and the poorest. Finally, the empirical application estimates that when subsistence needs are accounted for, the CBA-based desirability of a project decreases by 5% to 45% depending on the nature of the non-market component, with an average of 24%.

Our principal contribution is to the methodology used in CBA in presence of nonmarket components. We propose a framework for assessing the impact of subsistence needs on individual WTP elicited for non-market components, and a way to correct for this impact. This correction is added to the standard distributional weights, generally applied in CBA on ethical grounds. It also addresses certain methodological issues previously identified in stated preference elicitation (insensitivity to scale or scope, influence of question order in multiple-question elicitation, sensitivity of WTP to income, discrepancies between WTP and willingness to accept). Our contribution enables decision-makers employing CBA to better respect the preferences of the entire population with regard to non-market components.

The remainder of the paper proceeds as follows. Section 2 briefly sketches the equity issues in CBA. Section 3 describes the relevant models and proposes a correction factor. Section 4 analyses the impacts of introducing subsistence needs on the outcome of CBA: a desirability effect, a plutocratic bias and ranking reversal. Section 5 presents an empirical analysis on French income data. A discussion and conclusion are contained in Section 6.

2 CBA and equity issues

The century-long history of CBA has included its use by various national government agencies, supra-national organisations and private firms to assess the effectiveness of policies and prioritise them. CBA was gradually extended to all economic sectors (Swenson, 2015): beginning with navigation in the 1900s, CBA was applied to agricultural and land issues during the New Deal and subsequently to public urban and transportation infrastructures after World War II, to social, educational and health issues in the 1960s, to occupational and environmental issues in the 1970s. CBA was used to assess the value of regulation / deregulation and of central government interventions in various economic sectors during the 1980s and 1990s and to help compute various public profitability / efficiency ratios in the 2000s. From then on, it has been widely used in all sectors to support public and private decision-making.

The standard project-assessment criterion in CBA is the Kaldor-Hicks compensation test, an extension of the Pareto test for projects involving losers. It states that if the net present value of a project (measured by the discounted sum of individual benefits and costs) is positive, then social welfare is increased and the gainers can compensate the losers with a monetary transfer (Johansson and Kriström, 2018). If there are several projects that pass the compensation test, the decision-maker with limited economic resources is supposed to rank them according to the highest net present value, to maximize social welfare.

In early applications of CBA (Mishan, 1976; Harberger, 1978), individual costs and benefits were simply summed up. The rationale was that CBA targets efficiency (by allocating public funds) and not distributional issues, which are handled by the tax-transfer system (implicitly assuming that the tax-transfer system was "optimal" before the project). Arguments against this rationale rapidly emerged (Layard, 1980; Squire, 1980) in response to equity and distributional issues: in fact, there is no guarantee that the transfer between gainers and losers will actually take place once the project is implemented. It is commonly accepted today that distributional weights should correspond to the social marginal utility of income a_i , i.e. society's valuation of the individual marginal utility of income (Mäler, 1974; Kanninen and Kriström, 1992; Brent, 2006; Fleurbaey and Abi-Rafeh, 2016; Johansson and Kriström, 2018).

The change in social welfare associated with a project (dW), as measured with individual WTP (some of which may be negative), is then generally expressed as (Johansson and Kriström, 2018; Fleurbaey et al., 2013):¹

$$dW = \sum_{i} a_i \cdot WTP_i = \sum_{i} W_i \cdot V_i \cdot WTP_i$$

where W_i is the weight attributed to individual *i* in the social welfare function (i.e. the derivative of the social welfare function with respect to the utility level of that individual), and V_i is the individual's marginal utility of income (the derivative of the utility level of individual *i* with respect to her income). The social marginal utility of income a_i for individual *i* is the product of W_i by V_i , which is clearly an additional drawback to using constant a_i : it implies that W_i is the reciprocal of V_i , giving the poorest the lowest weight in the social welfare function, hence an anti-egalitarian distribution (Hau, 1987).

When non-market components are part of the change in social welfare, subsistence needs can limit individuals' ability to pay: irreducible and often pre-incurred costs limit the amount of income that can be freely spent on other goods or services, in particular non-market components. Brent (2006) notes that WTP "is dependent not only on their preferences but also on their ability to pay" and concludes that "WTP should be weighted", without indicating any specific procedure. We can use distributional weights a_i , but do they properly account for the ability to pay?

According to Fleurbaey et al. (2013), there are two main approaches available to estimate distributional weights: welfarist and non-welfarist. The welfarist approach estimates V_i and W_i separately. Whereas the former only depends on the characteristics of the individual utility function, the latter depends in addition on the choice of the social welfare function SW(.), with a parameter that stands for aversion to inequality. Among

¹ We restrict to small public projects unable to markedly change relative prices of goods or real income, to avoid inconsistencies (Brent, 2006; Boadway, 1974; Blackorby and Donaldson, 1990).

the many, generally non-independent, sources of inequalities in a population, income and (to a lesser extent) health status are those most frequently taken into account. To avoid choosing SW(.), practitioners sometimes directly specify the social marginal utility of income W_i , generally a function of income only (for instance, an adjustment of individual WTP by the ratio of individual income to average income, and the application of a power function to this ratio to account for inequality aversion). However, there is no consensus on how to set the power parameter (Brent, 2006; Johansson and Kriström, 2018).

The non-welfarist approach directly estimates the a_i , which requires interpersonal comparison of utilities (Adler, 2016; Fleurbaey and Abi-Rafeh, 2016). This can be achieved through the equivalent income approach based on the fairness principle: survey respondents are asked to give their preferences on different combinations of resources and a_i is measured directly in monetary terms (Fleurbaey et al., 2013; Samson et al., 2018). The subjective well-being approach can also be used to compare individuals' emotions, feelings or life satisfaction and derive distributional weights (see Fleurbaey and Abi-Rafeh, 2016; Bronsteen et al., 2013; for a discussion). Although non-welfarist approaches rely on surveys designed to elicit a_i separately from WTP_i , and are truly based on individual preferences, decision-makers almost never implement them in practice due to their complexity and the additional data required.

Overall, whatever the method used to determine a_i , the constraint from subsistence needs on ability to pay is not accounted for. This means that applying distributional weights to assess changes in utility for a project involving non-market components may not properly capture the varying ability to pay that is embedded in individual WTP. An additional weight is required to obtain subsistence-needs-corrected WTP for non-market components.

3 Models

Below, we detail the standard model and the model incorporating subsistence needs, determine the shadow prices based on a Constant Elasticity of Substitution (CES) utility function and propose a subsistence-needs correction factor.

3.1 Standard model

Consider an individual whose preference relation is continuous, monotonic and convex. Let us consider that this preference relation is represented by a two-good utility function u(x,q), where x represents the quantity of a composite market good and q the quantity of a non-market good. u(x,q) is assumed to be twice continuously differentiable, strictly increasing and strictly quasi-concave in x and q, with $x, q \ge 0$.

We are interested in the relation between the WTP for the non-market good and the individual's income (y). We follow the main trend in the literature on non-market valuation (Hanemann, 1991; Lankford, 1988; Ebert, 2003) by defining the marginal WTP as the shadow price for q:

$$\pi(p,q,y) = \frac{\partial v(p,q,y)/\partial q}{\partial v(p,q,y)/\partial y}$$
(1)

where v(p, q, y) is the indirect utility function obtained from the following maximization problem:

$$\operatorname{Max}_{x} u(x,q) \text{ subject to } px = y \text{ and } q \text{ fixed}$$
(2)

In this problem, the individual pays for a given quantity of non-market good q at shadow price π , has shadow income $y + \pi q$ and q is considered exogenous to the individual (see Horowitz et al., 2013; Dupoux and Martinet, 2018). The individual's income is compensated so that all his/her real income is spent on market goods. We then have the following equivalence with the inverse Hicksian demand: $\pi(p, q, y) = \pi(p, q, v(p, q, y))$. It is clear from Eq. (1) that the shadow price of the non-market good only depends on the individual's preferences represented by the parameters characterising v(.), quantity of non-market good q and income y.

3.2 Model with subsistence needs

Let us consider that individuals face minimum subsistence needs defined by level of consumption x_s of the composite good. Geary (1950) and Stone (1954) introduced these models to account for minimum levels of consumption in functions known as Stone-Geary functions. Since then, they have been adopted in general equilibrium models both at individual and at national level to account for the minimum agricultural production necessary for survival or exportation. More recently, Heal (2009) used this approach to address a subsistence requirement in terms of environmental / ecosystem services. In the following, we consider minimum subsistence needs to meet physiological requirements (see Maslow, 1943) and introduce these subsistence needs into u as follows:

$$\begin{cases} u(x,q) = u^{l}(x) & \text{for} \quad x \le x_{s} \\ u(x,q) = u^{h}(x,q) & \text{else} \end{cases}$$
(3)

We assume that u^l is strictly increasing in x and that u^h has the same properties as u (see above). We also adopt Baumgärtner et al. (2017b)'s assumption that individuals always prefer to be in the domain where subsistence needs are satisfied, i.e.:

$$\inf_{x > x_s, \ q \ge 0} u^h(x,q) > \sup_{x_s \ge x \ge 0} u^l(x) \tag{4}$$

Hence, below minimum subsistence needs, nobody is willing to trade the composite good for the non-market good, and π is consequently also set to 0. We will no longer consider this case in what follows. Above minimum subsistence needs, π is defined as in the standard model but based on the $u^h(x,q)$ function that accounts for these subsistence needs.

3.3 Defining the two models under a CES utility function

To determine the explicit relationships between the standard and the subsistence needs frameworks, we need to start from a functional form that is as flexible as possible regarding preferences. A relevant, easy to interpret, tractable and well-known specification of the utility function in consumer theory is the CES function first proposed by Arrow et al. (1961). For n goods, it is:

$$u(x_j) = \sum_{j} [\alpha_j x_j^{\theta}]^{\frac{1}{\theta}} \text{ for } \theta \in]-\infty; 1] ; 0 < \alpha_j < 1 ; \sum_{j} \alpha_j \equiv 1 ; j = 1, ..., n$$
 (5)

where x_j is the quantity of good j, α_j its share parameter and $1/(1-\theta)$ the elasticity of substitution. Note that the CES function covers a range from perfect complement $(\theta \to -\infty)$ to perfect substitute $(\theta \to 1)$, as well as the Cobb Douglas function $(\theta \to 0)$.²

Case of one non-market good

With the previous notations, the preferences over x and q are expressed in a CES as follows:

$$u(x,q) = \left[\alpha x^{\theta} + (1-\alpha)q^{\theta}\right]^{\frac{1}{\theta}} \text{ for } \theta \in \left] -\infty; 1\right] ; 0 < \alpha < 1$$
(6)

where α represents the preference for market good x relative to the preference for non-market good q.

Since we consider that an individual spends all his/her income on market goods (see maximization problem (2)), we set $x \equiv y/p$ and $p \equiv 1$ in the following. Thus, composite market good x can be seen as the numéraire representing the individual's income. At

² Indeed, $u(x_j) \to \prod_j x_j^{\alpha_j}$ when $\theta \to 0$.

equilibrium, the shadow price for the non-market good in the standard CES model is (from Eq. (1) and Eq. (6)):

$$\pi = \frac{(1-\alpha)q^{\theta-1}}{\alpha x^{\theta-1}} \tag{7}$$

We extend the Stone-Geary function to CES (see Baumgärtner et al., 2017b; Drupp, 2018), and define the extended CES above minimum subsistence level as follows:

$$u^{h}(x,q) = [\alpha(x-x_{s})^{\theta} + (1-\alpha)q^{\theta}]^{\frac{1}{\theta}} \quad \text{for} \quad x > x_{s}; \quad \theta \in]-\infty;1]; \ 0 < \alpha < 1$$
(8)

The shadow price for q in the CES model with subsistence needs is (from Eq.(1) and Eq.(8)):

$$\pi_s = \frac{(1-\alpha)q^{\theta-1}}{\alpha(x-x_s)^{\theta-1}} \tag{9}$$

It is clear that π_s is always lower than π , except when $x_s = 0$, in which case $\pi_s \equiv \pi$.

Case of two non-market goods

We extend the individual CES function in Eq. (6) to the case of two non-market goods, with quantity q_1 and q_2 :

$$u(x,q_1,q_2) = \left(\alpha_0 x^{\theta} + \alpha_1 q_1^{\theta} + \alpha_2 q_2^{\theta}\right)^{1/\theta}$$

with α_0 the preference for the composite market good, α_1 and α_2 respectively the preferences for the first and second non-market goods, and $\alpha_0 + \alpha_1 + \alpha_2 \equiv 1$. The shadow prices for the two non-market goods are respectively:

$$\pi_1 = \frac{\alpha_1}{\alpha_0} \left(\frac{q_1}{x}\right)^{\theta-1} \quad \text{and} \quad \pi_2 = \frac{\alpha_2}{\alpha_0} \left(\frac{q_2}{x}\right)^{\theta-1}.$$
 (10)

Extending the Stone-Geary function similarly to Eq. (8) leads to:

$$\pi_{1,s} = \frac{\alpha_1}{\alpha_0} \left(\frac{q_1}{x - x_s}\right)^{\theta - 1} \quad \text{and} \quad \pi_{2,s} = \frac{\alpha_2}{\alpha_0} \left(\frac{q_2}{x - x_s}\right)^{\theta - 1}.$$
 (11)

As with one non-market good, $\pi_{.,s}$ is always lower than $\pi_{.}$, except when $x_s = 0$.

3.4 Deriving a subsistence-needs correction factor

Does the use of an *ex post* distributional weight based on the social marginal utility of income (see Section 2) properly correct for the effect of subsistence needs? For an individual i with preferences (α_i, θ_i) and given one non-market good, the marginal utility of income V_i is equal to:

$$V_i = \frac{\partial u(x_i, q, x_s)}{\partial x_i} = \frac{\partial [\alpha_i (x_i - x_s)^{\theta_i} + (1 - \alpha_i)q^{\theta_i}]^{\frac{1}{\theta_i}}}{\partial x_i}$$

$$= \left[\alpha_i (x_i - x_s)^{\theta_i} + (1 - \alpha_i)q^{\theta_i}\right]^{\frac{1 - \theta_i}{\theta_i}} \alpha_i (x_i - x_s)^{\theta_i - 1}$$

If we weight the shadow prices in the model with subsistence needs (Eq. (9)) by the marginal utility of income, we obtain:

$$\pi_{w_{i,s}} = V_i \pi_{i,s} = \left[\alpha_i (x_i - x_s)^{\theta_i} + (1 - \alpha_i) q^{\theta_i} \right]^{\frac{1 - \theta_i}{\theta_i}} (1 - \alpha_i) q^{\theta_i - 1}$$

The resulting shadow price $\pi_{w_{i,s}}$ still depends on the level of subsistence needs x_s : accounting for the marginal utility of income does not fully account for the effect of subsistence needs. Indeed, the appropriate individual subsistence-needs correction factor Z_i should make the shadow price $\pi_{i,s}$ equal to a shadow price π_i in a situation where $x_s = 0$, i.e. where subsistence needs are borne by others. Consequently, we obtain from identity $Z_i \pi_{i,s} = \pi_i$:

$$Z_{i} = \frac{\pi_{i}}{\pi_{i,s}} = \frac{\frac{(1-\alpha_{i})q^{\theta_{i}-1}}{\alpha_{i}x_{i}^{\theta_{i}-1}}}{\frac{(1-\alpha_{i})q^{\theta_{i}-1}}{\alpha_{i}(x_{i}-x_{s})^{\theta_{i}-1}}} = \left(\frac{x_{i}-x_{s}}{x_{i}}\right)^{\theta_{i}-1}$$

Note that Z_i does not depend on the preferences for the market good α_i . In the case of two non-market goods, it is clear from Eq. (10) and (11) that Z_i is also defined as above for each non-market good. In a population of individuals with different incomes and preferences, the average subsistence-needs correction factor E(Z) is:

$$E(Z) = \frac{\sum_{i} \pi_i}{\sum_{i} \pi_{i,s}}$$

The correction factor is to be separated from the distributional weights (based on ethical principles to account for health, economic or social inequalities), since it is only meant to correct for the impact of subsistence needs on ability to pay for non-market goods. A change in social welfare associated with a project involving non-market components is then:

$$dW = \sum_{i} W_i \cdot V_i \cdot Z_i \cdot \pi_{i,s} = \sum_{i} a_i \cdot Z_i \cdot \pi_{i,s}$$

Overall, omitting this correction factor Z_i affects the assessment of benefits. It not only decreases the desirability of a project at a given cost, but also has an impact on project ranking that differs depending on whether the population's preferences for the non-market good (measured by θ and α) are homogeneous or heterogeneous over income groups. This is explored in the next section.

4 Impacts on CBA assessment

We explore how incorporating subsistence needs into a CBA impacts the shadow prices of the non-market good as well as the outcomes. Consequently, based on Eq. (7) and (9), we consider how the subsistence-needs correction factor Z_i evolves when income changes, simplifying interpretation by removing the price-level effect. The closer Z_i is to 1, the smaller the correction required to account for subsistence needs. We consider first how project desirability as assessed by CBA is affected when preferences for one project are homogeneous over individuals (i.e. same α) and characterize the plutocratic bias when they are heterogeneous between income groups (i.e. different α). Then we consider how relative CBA outcomes for two projects (i.e. project rankings) are affected when preferences are heterogeneous.

4.1 Decreased project desirability

If preferences are homogeneous over the population, ignoring the minimum subsistence level leads to under-estimating the benefits measured with shadow prices, which decreases the overall desirability of a project at a given cost. The size of this decrease will depend on the actual distribution of income in the population (see an application in section 6). Figure 1 illustrates the magnitude of correction factor Z_i for different combinations of θ and income (represented in terms of x/x_s). It can be seen that the lower the income, the higher Z_i for any given value of θ , due to the fact that π_s tends towards zero when income tends towards x_s . For an income seven times larger than x_s , for instance, the correction factor is about 1.26 (i.e. a 26% increase in WTP) when θ exhibits complementarity ($\theta = -0.5$), but only about 1.08 for substitutability ($\theta = 0.5$). This means that, at the aggregate level, the desirability of projects with a non-market component will be underestimated if correction factor Z_i is not applied to account for subsistence needs. This deprives the population of projects that would potentially generate a social benefit greater than their costs.

4.2 A plutocratic bias

If preferences regarding the non-market good are heterogeneous (i.e. vary over income groups), the overall desirability of a project is still lower when the subsistence needs issue is ignored. In addition, relative desirability too appears to be biased towards the preferences of the richest: omitting correction factor Z_i has a greater downward impact on shadow prices for the poorest fraction of the population than for the richest.

As an illustration, imagine a population with a bi-modal income distribution (for the sake of simplicity): M individuals have a low income ($x^L = 1.5$ times subsistence needs), and N individuals have a high income ($x^H = 10$ times subsistence needs).³ Let

 $^{^{3}}$ The generalisation to more income groups in a discrete or continuous way is straightforward but would



Figure 1: Correction factor Z_i when preferences are homogeneous (by θ and income relative to x_s)

us consider that the preferences of the two subpopulations regarding the non-market good are respectively measured by α^L for the poorest and α^H for the richest, varying by stepsize .05, from .05 (strong preference for the non-market good) to .95 (strong preference for the market good). For each combination of preferences, the corrections for each group of income are Z^L, Z^H , and the average correction factor in the population is $E(Z) = (M + N)^{-1}[MZ^L + NZ^H]$. This indicates the difference that can be expected in the valuation of non-market benefits between a CBA based on elicited preferences subject to the ability-to-pay constraint and a CBA based on elicited preferences with no constraint. The closer to one, the lower the impact of ability to pay on the population's non-market preferences. We compute E(Z) for three values of substitutability ($\theta = 0.5, 0, -0.5$), and give the results in Figure 2, for M = N.

Whatever the substitutability value θ , we have the following results. For homogeneous preferences (represented by the diagonal black line segment on the three figures), the correction factor is clearly constant, with a spread of about 20%. Provided the preferences of the richest are more non-market oriented than those of the poorest (to the left of the diagonal), E(Z) is lower than in the homogeneous case. The preferences of the richest (for

make the graphic representation less intuitive.

the non-market good) are favoured, because the shadow price for the non-market good requires less correction to properly account for ability to pay. Provided the preferences of the richest are less non-market oriented than those of the poorest (to the right of the diagonal), E(Z) is higher than in the homogeneous case. The preferences of the richest (now for the market good) are favoured again, because more correction is required. Note that when the number of the poorest differs from the number of the richest, the previous results still hold, the average correction factor being higher when M > N and lower when N > M, ceteris paribus.

Overall, whatever the preferences of the richest, they are always better represented in CBA than those of the poorest, a kind of plutocratic bias. In particular, the non-market preferences of the poorest appear never to be properly accounted for unless shared by the richest (i.e. the homogenous case), whereas the non-market preferences of the richest are unfailingly better accounted for, for two reasons. First, their shadow prices are higher, as seen in section 3.3. Second, their shadow prices are also less underestimated: the parts of Figure 2 to the left of the diagonal are always closer to 1 than those to the right. Applying the correction factor Z_i removes this plutocratic bias.

4.3 Potential ranking reversal

We now look at how CBA-based ranking of two projects can be affected by incorporating subsistence needs when these projects have two different non-market components. We study the condition for ranking reversal, based on the shadow prices defined in section 3.3 for two non-market goods.

If preferences are homogeneous among income groups, there cannot be ranking reversal by definition, because everyone will prefer the same project. However, in the heterogeneous case, there may be ranking reversal between projects depending on whether their desirability is assessed with or without the subsistence-needs correction factor. We consider two projects (A and B)⁴ and introduce heterogeneity in preferences for non-market goods as before, by considering a population composed of N individuals with a high income (with preferences α_A^H and α_B^H) and M individuals with a low income (α_A^L and α_B^L , with $\alpha_0 + \alpha_A^H + \alpha_B^H = 1$ and $\alpha_0 + \alpha_A^L + \alpha_B^L = 1$).

 $^{^{4}}$ To avoid adding complexity, we assume that the non-market components of the two projects are different. Otherwise, a function accounting for the proportion of the two components in each project would be required.

Figure 2: Average correction factor when preferences are heterogeneous (upper panel: $\theta = .5$, middle panel: $\theta = 0$, lower panel: $\theta = -.5$)



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Ranking reversal only occurs if the following condition is fulfilled (see proof in Appendix A):

$$1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H - x_s}{x^L - x_s} \right)^{1 - \theta} (1 - \alpha_0 - 2\alpha_A^H) > 2\alpha_B^L > 1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1 - \theta} (1 - \alpha_0 - 2\alpha_A^H)$$

Figure 3 (and 4 and 5 in Appendix B) represents this condition in terms of α_A^H and α_B^L for three values of θ , and as a function of α_0 . For convenience and ease of comparison with previous results, we set as previously x^L at 1.5 times subsistence needs, and x^H at 10 times subsistence needs. On the X-axis, we measure α_A^H , the preference for project A of the high-income individuals: the closer to $(1 - \alpha_0)$, the stronger this preference, whereas the closer to 0, the stronger the preference for project B, while $(1 - \alpha_0)/2$ indicates indifference between the two projects (i.e. $\alpha_A^H = \alpha_B^H$). On the Y-axis, we measure α_B^L , the preference for project B of the low-income individuals: the closer to $(1 - \alpha_0)$, the stronger the stronger the individuals the closer to $(1 - \alpha_0)$, the stronger the individuals the closer to $(1 - \alpha_0)$, the stronger this preference for project A, with indifference at $(1 - \alpha_0)/2 = \alpha_A^L = \alpha_B^L$.

Each figure is composed of four quadrants. Interpretation of the figures does not depend on θ , nor on the income distribution.

In quadrants 1 and 4, no ranking reversal is observed since individuals with both low and high incomes prefer the same project (project B in quadrant 1 (cases A-Ia and B-IIa in Appendix A) and project A in quadrant 4 (cases A-IIa and B-Ia in Appendix A)), although to varying degrees.

In quadrant 2, the low-income individuals prefer project B while the high-income individuals prefer project A (the 45° line represents a similar respective degree of preference). The area above the ranking reversal area represents the pattern arising from the combined preferences of high- and low-income individuals: a preference for project B. The area below it represents the pattern of preference for project A. The ranking reversal area represents combinations of preferences for which project A is preferred based on elicited WTP whereas project B is preferred based on subsistence-needs-corrected WTP (case B-Ib in Appendix A): the preferences of high-income individuals are favoured in the absence of correction. Quadrant 3 shows a similar situation, with low-income individuals preferring project A and high-income individuals project B. The ranking reversal area now represents a pattern of preference for project B based on elicited WTP and preference for project A based on subsistence-needs-corrected WTP (caseA-Ib in Appendix A): the preferences of high-income individuals project B based on elicited WTP and preference for project A based on subsistence-needs-corrected WTP (caseA-Ib in Appendix A): the preferences of high-income individuals are again favoured in the absence of correction.

Figure 3 represents the case where $\theta = 0.5$ for different income distributions in the

population (M = N, M = 3N and N = 3M).

We observe that when M = N, the area of ranking reversal is below the 45° line in quadrant 3 (and above it in quadrant 2), which confirms the previous finding that highincome individuals' preferences (for project B in quadrant 3 and project A in quadrant 2) dominate those of low-income individuals (for project A in quadrant 3 and project B in quadrant 2) when WTP is not corrected for subsistence needs, i.e. there is a plutocratic bias.

When there are three times more low incomes than high incomes (M = 3N), areas of the poorest individuals' preferred project grow and cross the 45° line: the number effect makes it more likely that their preferred project will be chosen, although the subsistence-needs effect still favours the richest. There is still a ranking reversal area, which grows.

Finally, when there are three times more high incomes than low incomes (3M = N), the number effect reinforces the dominance of the richest individuals' preferences so that there is almost no room for the preferences of the poorest, and ranking reversal areas, albeit limited, are still observed.

The cases where $\theta = -0.5$ (complementarity) and $\theta = 0$ are shown in Appendix B. All previous results hold. A decrease in θ - the substitutability between the composite market good and the non-market goods - leads to decreased likelihood of the poorest individuals' preferences dominating, as well as to smaller ranking reversal areas. The preferences of the richest become increasingly likely to dominate as substitutability decreases and/or their proportion in the population increases.

Overall, we find that neglecting the subsistence-needs correction factor when a population's preferences are heterogeneous creates the conditions for ranking reversal between two projects, always with a bias towards the richest individuals' preferences, whatever they are.

5 Empirical analysis on French income data

We use real data to examine the extent to which CBA might be affected by the subsistence needs issue in real life. We rely both on French income distribution data and on empirical studies eliciting elasticities of substitution for various non-market goods.

Regarding the distribution of income, we first have to set the cost of meeting subsistence needs x_s . This is different from absolute monetary poverty, defined by the World

Figure 3: Pattern of preferences leading to ranking reversal between two projects when $\theta = .5$ (upper panel: N = M, middle panel: M = 3N, lower panel: N = 3M)



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Bank based on a minimum number of calories (costing about US\$ 1.9 in 2015). It is also different from relative monetary poverty, which takes into account the distribution of income in a given society. The Organisation for Economic Co-operation and Development (OECD) (as well as Institut National de la Statistique et des Etudes Economiques, in France) consider for instance that households with a standard of living⁵ lower than 50% of the median standard of living of the population are below the poverty threshold, whereas Eurostat sets the threshold to be at 60% of the median. What we are looking for is the minimum amount required to live in a given country, including minimum expenditure on food, water, energy, housing, clothes, transportation, etc. A French survey estimates this to be about \in 600 per month for one adult in 2016 (Carrefour des Solidarités, 2011). This amount is slightly lower than the median French standard of living for a single person considered poor (€ 705 per month in 2015, Argouarc'h and Cazenave-Lacrouts, 2017) and slightly higher than the active solidarity revenue (RSA) paid by the French government to a single individual with no resources (\in 545 per month in 2017). We therefore adopt as a reasonable benchmark an annual cost of meeting subsistence needs per individual of $x_s \in 600 \times 12 = \notin 7200$. The French distribution of annual standard of living by unit of consumption (Institut National de la Statistique et des Etudes Economiques, 2017) is expressed in terms of subsistence needs in the first three columns of Table 5.

Regarding the substitutability of market goods for non-market goods, Drupp (2018) gathers θ estimates for various non-market services (air or water quality improvements, forest or marine services, landscape or recreational amenities, biodiversity, etc.) in different countries. He computes a mean empirical estimate of $\theta = 0.57$ with a mean empirical error range of (0.28 - 0.86). This means that, on average, individuals exhibit substitutability between market goods and non-market goods or services. According to our results in section 4, the subsistence-needs correction factor would be lower *ceteris paribus* than for complementarity, hence the preferences of the poorest would be better accounted for, although there is room for ranking reversal between projects.

We assume that preferences regarding the non-market good are homogeneous (over income groups), due to lack of relevant data on their actual distribution w.r.t. income in the population. Table 5 represents the subsistence-needs correction factor Z for various values of θ and various levels of standard of living (remember from section 3, that this

 $^{^{5}}$ These bodies use the OECD-modified scale to calculate equivalised disposable income / standard of living per unit of consumption. This scale assigns a value of 1 to the household head, of 0.5 to each additional adult member and of 0.3 to each child below 14.

Standard of	Ratio	Fraction of				θ			
living $(\in 10^3)$	x/x_s	population	0.9	0.6	0.3	0	-0.3	-0.6	-0.9
7.2	$1(x_{s})$	0.0%	1.699	8.342	40.95	201.0	986.6	2500	5200
10.8	$1.5 \; (x^L)$	11.6%	1.175	1.904	3.085	5.000	8.103	13.16	21.28
14.4	2	10.4%	1.088	1.403	1.810	2.333	3.009	3.876	5.000
18	2.5	18.7%	1.061	1.265	1.509	1.800	2.147	2.564	3.058
21.6	3	14.5%	1.046	1.198	1.372	1.571	1.800	2.062	2.358
25.2	3.5	13.4%	1.037	1.158	1.294	1.444	1.613	1.802	2.012
28.8	4	10.8%	1.032	1.132	1.242	1.364	1.497	1.642	1.802
32.4	4.5	4.8%	1.027	1.113	1.207	1.308	1.417	1.536	1.664
36	5	2.9%	1.024	1.099	1.180	1.267	1.360	1.460	1.567
39.6	5.5	2.1%	1.021	1.088	1.159	1.235	1.316	1.403	1.495
43.2	6	1.7%	1.019	1.079	1.143	1.211	1.282	1.357	1.437
50.4	7	2.6%	1.016	1.066	1.119	1.174	1.232	1.292	1.357
57.6	8	2.0%	1.014	1.057	$1,\!102$	$1,\!148$	$1,\!197$	1,247	1.300
64.8	9	1.6%	1.012	1.050	1.089	1.129	1.171	1.214	1.259
72	$10 \; (x^H)$	1.4%	1.011	1.044	1.079	1.114	1.151	1.189	1.229
72 and over	—	1.8%	1.010	1.040	1.071	1.103	1.135	1.178	1.215
Weighted average		100%	1.058	1.241	1.437	1.648	1.875	2.119	2.381

Table 1: Subsistence needs correction factor Z in the French population, by θ

difference does not depend on α in the homogeneous case). In particular, when $\theta = 0.57$ (respectively (0.28 - 0.86)), the income-weighted average correction factor E(Z) is about 1.24 (respectively 1.44 and 1.06). This means that, from a CBA perspective, the non-market benefits of a project (assessed through survey-based shadow prices) are under-estimated by 24% on average compared to those obtained when subsistence needs are accounted for. Consequently, because of the distortion of the cost-benefit test (benefits need to be 24% greater than costs for a project to pass), this rules out a portion of socially desirable projects. In addition, heterogeneous preferences over income groups may add a plutocratic bias towards the preferences of the richest that can lead to ranking reversal when more than one project is proposed. Due to lack of specific data on the population's distribution of preferences for the non-market good w.r.t. income, we cannot currently assess the extent of these effects.

6 Discussion

Our findings show that the desirability of a project containing a non-market component is always driven by the preferences of the richest when assessed with standard CBA. This is true not only in absolute terms, but also in relative terms when the population's preferences differ. Incorporating the subsistence-needs correction factor ensures that the preferences of the poorest too are taken into account, without precluding the additional use of distributional weighting to tackle the equity and distributional issues. In many CBAs, differences in the non-market values chosen to assess welfare change may lead to divergent conclusions (see for instance Sterner and Persson, 2008; on the economic impact of climate change). Our findings have implications for some methodological aspects of preference elicitation and for empirical research in economics, in addition to raising policy issues.

From a methodological perspective, what most stated and revealed preference studies expect to find is a positive and significant relationship between income and WTP. This relationship indicates that individuals are behaving as they would on actual marketplaces, where level of income drives level of consumption. Our framework suggests that this validity criterion for preference revelation also reflects a constrained expression of the preferences of the poorest for a non-market good or service. Corroborating evidence is found in Olsen et al. (2005), for instance, who found that a substantial proportion of individuals showing indifference between two programs when assessed through their WTP show a clear ordinal ranking when assessed with ordinal preferences.

In addition, it may be worth accounting for subsistence needs when studying the scope / scale effect in contingent valuation. Smith (2005) was interested in the role of the budget constraint in the scale-sensitivity of WTP. He found "an increasing 'relevance' of the budget constraint as the value of the good (relative to income) increases: as the benefit increases, WTP for that benefit rises and consequently the budget constraint becomes an increasingly significant determinant of WTP". In cases where WTP is found not proportional to the scope or the scale of an improvement proposed in a survey, at least part of the reason may be the constraint that subsistence needs impose on WTP. This constraint is not only more binding for the poorest individuals than for the richest, but is also heightened when the scale / scope of the valuation is large.

In the same vein, Breffle et al. (2015) found that lower-income individuals could not afford a second program required to fully improve a recreational site. In surveys like theirs, where multiple elicitation questions are used, the subsistence-needs correction factor we propose can be made dependent on the elicitation round. Based initially on $(x - x_s)$ when WTP is elicited in round 1, x_s would progressively be supplemented with the WTPs given in the previous rounds, thus increasing the correction factor as income net of pre-incurred expenses (subsistence needs plus the successive WTPs) decreases.

From an empirical perspective, our findings challenge the use of stated and revealed

preferences techniques in CBA involving preferences for a non-market component and measured through WTP. When these preferences are homogeneous, although the benefits associated with a project are under-estimated, the average preference of the population is properly accounted for. When these preferences depend on income groups (or any other sociodemographic characteristics correlated with income, like education, place of residence or age), the preferences of the poorest individuals may be under-represented and those of the richest over-represented. Decision-makers should be aware of the plutocratic bias and the possibility of project ranking reversal.

Several *ex ante* methods that better account for the non-market good preferences of the whole population already exist. If preferences are elicited through willingness to exchange money, there are normalized scenarios involving a given (hypothetical) income assumed equal for everyone (Chanel et al., 2013). Money can also be used to buy votes for or against a project (Masur, 2017). Alternatives to money include contributions in kind or in work (Brouwer et al., 2009; Abramson et al., 2011; Hossack and An, 2015). Instead of using a numéraire, a simple ranking of projects (Olsen et al., 2005) providing ordinal information would suffice to remove the income and subsistence issues but would be of limited policy use. An interesting future avenue of research would be to compare the results of these *ex ante* methods with those obtained with the *ex post* correction factor we propose.

In terms of policy implications, our findings raise important issues. They contribute to the equity vs. equality debate around whether WTP elicited from the poorest should be corrected when preferences for a non-market good are known to depend on income groups. Doing so means favouring equity (an as-fair-as-possible representation of preferences) over equality (a common representation of preferences). How would public priorities be affected (see Olsen and Donaldson, 1998; for a discussion regarding health, where priorities depend on income or age)? What impact would a change in income inequality have on CBA assessment? In a recent paper, Baumgärtner et al. (2017a) show that, depending on whether the non-market good is a substitute or a complement for manufactured goods, increased inequality can either decrease or increase mean WTP. Once non-market values are involved in a CBA, we need to be on the lookout for any potential plutocratic bias or ranking reversal effect.

Finally, our work is subject to limitations. First, for the sake of interpretation and tractability, we chose to derive our subsistence-needs correction factor from one specific utility function. The CES was the best candidate: an absolutely standard and flexible

function widely used in consumer theory that fits very different patterns of preferences. Second, what we propose is a way to properly account for the non-market preferences of the whole population when comparing several projects, within the CBA framework. However, we are aware that there are issues with using our method to assess whether one particular project will indeed pass the cost-benefit test once implemented. Actually, if the project is to be funded through individual contributions, there is no guarantee that the poorest will be able to contribute as much as they indicate through the subsistence-needs-corrected WTP.

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A Appendix A Condition for ranking reversal

Ranking reversal occurs when the relative ranking of 2 projects (A and B) depends on the way they are evaluated (via elicited WTP or to subsistence-needs corrected WTP). We use the notations previously introduced.

A] Consider that project A is ranked first when subsistence needs are properly accounted for (condition A1) but ranked second based on elicited WTP (condition A2), i.e.: $(A1): N\pi_{s,A}^{H} + M\pi_{s,A}^{L} > N\pi_{s,B}^{H} + M\pi_{s,B}^{L}$ and $(A2): N\pi_{B}^{H} + M\pi_{B}^{L} > N\pi_{A}^{H} + M\pi_{A}^{L}$

A-I] Assume that $\alpha_A^H < \alpha_B^H$. A-Ia] Assume that $\alpha_A^L \le \alpha_B^L$.

Ranking reversal is meaningless, as both income groups prefer project B.

A-Ib] Assume that $\alpha_A^L > \alpha_B^L$.

When ranking is based on subsistence-needs-corrected WTP, condition (A1) leads to (from Eq. (11)):

$$N\frac{\alpha_A^H}{\alpha_0} \left(\frac{q}{x^H - x_s}\right)^{\theta - 1} + M\frac{\alpha_A^L}{\alpha_0} \left(\frac{q}{x^L - x_s}\right)^{\theta - 1} > N\frac{\alpha_B^H}{\alpha_0} \left(\frac{q}{x^H - x_s}\right)^{\theta - 1} + M\frac{\alpha_B^L}{\alpha_0} \left(\frac{q}{x^L - x_s}\right)^{\theta - 1}$$

$$\iff N\alpha_A^H \left(x^H - x_s\right)^{1 - \theta} + M\alpha_A^L \left(x^L - x_s\right)^{1 - \theta} > N\alpha_B^H \left(x^H - x_s\right)^{1 - \theta} + M\alpha_B^L \left(x^L - x_s\right)$$

$$\iff M \left(x^L - x_s\right)^{1 - \theta} \left(\alpha_A^L - \alpha_B^L\right) > N \left(x^H - x_s\right)^{1 - \theta} \left(\alpha_B^H - \alpha_A^H\right)$$

Because $x^H > x^L > x_s$, we have:

$$\left(\frac{x^H - x_s}{x^L - x_s}\right)^{1-\theta} < \frac{M}{N} \frac{\alpha_A^L - \alpha_B^L}{\alpha_B^H - \alpha_A^H} \tag{12}$$

We know that $\alpha_A^L = 1 - \alpha_0 - \alpha_B^L$ and $\alpha_B^H = 1 - \alpha_0 - \alpha_A^H$, hence:

$$\left(\frac{x^H - x_s}{x^L - x_s}\right)^{1-\theta} < \frac{M}{N} \frac{1 - \alpha_0 - 2\alpha_B^L}{1 - \alpha_0 - 2\alpha_A^H}$$

Then, because $\alpha_A^H < \alpha_B^H$ and $\alpha_A^L > \alpha_B^L$, we have $1 - \alpha_0 - 2\alpha_A^H > 0$ and $1 - \alpha_0 - 2\alpha_B^L > 0$. Easy manipulations lead to:

$$\frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H-x_s}{x^L-x_s}\right)^{1-\theta}\left(1-\alpha_0-2\alpha_A^H\right)\right) > \alpha_B^L$$

When ranking is based on elicited WTP, and by the same reasoning, condition (A2) leads to (from Eq. (10)):

$$\frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\left(1-\alpha_0-2\alpha_A^H\right)\right) < \alpha_B^L$$

Consequently, conditions (A1) and (A2) taken together implies that:

$$\frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H - x_s}{x^L - x_s} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right) > \alpha_B^L > \frac{1}{2} \left(1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1-\theta} (1 - \alpha_0 - 2\alpha_A^H) \right)$$

hence the ranking reversal condition:

$$1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H - x_s}{x^L - x_s} \right)^{1 - \theta} (1 - \alpha_0 - 2\alpha_A^H) > 2\alpha_B^L > 1 - \alpha_0 - \frac{N}{M} \left(\frac{x^H}{x^L} \right)^{1 - \theta} (1 - \alpha_0 - 2\alpha_A^H)$$

AII] Assume that $\alpha_A^H > \alpha_B^H$.

AII-a] Assume that $\alpha_A^L \ge \alpha_B^L$.

Ranking reversal is meaningless, as both income groups prefer project A.

A-IIb] Assume that $\alpha_A^L < \alpha_B^L$

By the same reasoning as the in Ib], condition (A1) leads to:

$$\frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H-x_s}{x^L-x_s}\right)^{1-\theta}\left(1-\alpha_0-2\alpha_A^H\right)\right)<\alpha_B^L$$

and condition (A2) leads to:

$$\frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\left(1-\alpha_0-2\alpha_A^H\right)\right) > \alpha_B^L$$

Consequently, we have:

$$\frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H-x_s}{x^L-x_s}\right)^{1-\theta}\left(1-\alpha_0-2\alpha_A^H\right)\right) < \alpha_B^L < \frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\left(1-\alpha_0-2\alpha_A^H\right)\right) < \alpha_B^L < \frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\left(1-\alpha_0-2\alpha_A^H\right)\right) < \alpha_B^L < \frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\left(1-\alpha_0-2\alpha_A^H\right)\right) < \frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\right) < \frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\right) < \frac{1}{2}\left(1-\alpha_0-\frac{N}{M}\left(\frac{x^H}{x^L}\right)^{1-\theta}\right)$$

This condition cannot be met since $(1 - \alpha_0 - 2\alpha_A^H) < 0$ and $\left(\frac{x^H - x_s}{x^L - x_s}\right)^{1-\theta} > \left(\frac{x^H}{x^L}\right)^{1-\theta} \forall x^H, x^L, x_s, \theta$ in their respective previously-defined domains of definition.

A-III] Assume that $\alpha_A^H = \alpha_B^H$. Ranking reversal is meaningless: Equation (12) cannot be met since $\left(\frac{x^H - x_s}{x^L - x_s}\right)^{1-\theta} > 0$.

B] Consider that project *B* is ranked first when subsistence needs are properly accounted for (condition A3) but ranked second based on elicited WTP (condition A4), i.e.: $(A3): N\pi_{s,A}^{H} + M\pi_{s,A}^{L} < N\pi_{s,B}^{H} + M\pi_{s,B}^{L}$ and $(A4): N\pi_{B}^{H} + M\pi_{B}^{L} < N\pi_{A}^{H} + M\pi_{A}^{L}$ We use the symmetry argument to obtain the same valid ranking reversal condition as in A].

B-I] Assume that $\alpha_A^H > \alpha_B^H$.

B-Ia] Assume that $\alpha_A^L \ge \alpha_B^L$.

Ranking reversal is meaningless, as both income groups prefer project A.

B-Ib] Assume that $\alpha_A^L < \alpha_B^L$.

See A-Ib] for the proof of the ranking reversal condition.

B-II] Assume that $\alpha_A^H < \alpha_B^H$.

B-IIa] Assume that $\alpha_A^L \leq \alpha_B^L$.

Ranking reversal is meaningless, as both income groups prefer project B.

B-IIb] Assume that $\alpha_A^L > \alpha_B^L$.

Impossible (see A-IIb] for proof).

B-III] Assume that $\alpha_A^H = \alpha_B^H$.

Ranking reversal is meaningless (see A-III]).

B Appendix **B** Pattern of preferences leading to a ranking reversal between two projects when $\theta = 0$ and $\theta = -0.5$





Figure 5: $\theta = -0.5$ (complementarity).